6.004 Fall 2018 Tutorial Problems
L02 – Combinational Devices and Boolean Algebra

Problem 1.

Consider the truth table on the right, which defines two functions F and G of three input variables (A, B, and C).

For each function, write it in normal form, then find a minimal sum of products (minimal SOP) expression.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Normal form for F(A,B,C) = ________________________________

Minimal sum of products for F(A,B,C) = ________________________________

Normal form for G(A,B,C) = ________________________________

Minimal sum of products for G(A,B,C) = ________________________________
Problem 2.

Consider the 3-input Boolean function \( G(A,B,C) = \overline{A} \cdot \overline{C} + A \cdot \overline{B} + B \cdot \overline{C} \).

1. How many 1’s are there in the output column of G’s 8-row truth table?

2. Give a minimal sum-of-products expression for G.

3. There’s good news and bad news: the bad news is that the stockroom only has G gates. The good news is that it has as many as you need. Using only combinational circuits built from G gates, one can implement (choose the best response):
   
   (A) Any Boolean function (G is functionally complete)
   (B) Only functions with 3 inputs or less
   (C) Only functions with the same truth table as G

4. Can a sum-of-products expression involving 3 input variables with greater than 4 product terms always be simplified to a sum-of-products expression using fewer product terms?
Problem 3.

Consider the logic diagram shown below, which includes XOR2, NAND2, NAND3, and INV (inverter) gates.

![Logic Diagram]

<table>
<thead>
<tr>
<th>Gate</th>
<th>$t_{PD}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>INV</td>
<td>1.0 ns</td>
</tr>
<tr>
<td>NAND2</td>
<td>1.5 ns</td>
</tr>
<tr>
<td>NAND3</td>
<td>1.8 ns</td>
</tr>
<tr>
<td>XOR2</td>
<td>2.5 ns</td>
</tr>
</tbody>
</table>

1. Using the $t_{PD}$ information for the gate components shown in the table above, compute the $t_{PD}$ for the circuit.

$$t_{PD} = \phantom{123456789012345678901234567890}$$

2. Find minimal sum-of-products expressions for both outputs, $D$ and $Bout$.

NOTE: The gates implement the following functions:
- $NAND2(a, b) = \overline{a \cdot b}$
- $NAND3(a, b, c) = \overline{a \cdot b \cdot c}$
- $XOR2(a, b) = a \cdot \overline{b} + \overline{a} \cdot b$

Minimal sum of products for $D(X,Y,\text{Bin}) = \phantom{1234567890123456789012345678901234567890}$

Minimal sum of products for $Bout(X,Y,\text{Bin}) = \phantom{1234567890123456789012345678901234567890}$
Problem 4.

Simplify the following Boolean expressions by finding a minimal sum-of-products expression for each one:

1. \( ac + b + c \)
2. \( (a + b)c + \bar{c}a + b(\bar{a} + c) \)
3. \( a(b + \bar{c})(b + a(b + c)) \)
4. \( a(b + c(d + ef)) \)
Problem 5.

There are some Boolean expressions for which no assignment of values to variables can produce True (e.g., \( \overline{a} \overline{a} \)). Those Boolean expressions are said to be non-satisfiable. Are the following Boolean expressions satisfiable? If the expression is satisfiable, give an assignment to variables that makes the expression evaluate to True. If the expression is non-satisfiable, prove it.

1. \((a + b)c + \overline{c}a + b(\overline{a} + c)\)
2. \((x + y)(x + \overline{y})(z + \overline{y})(y + \overline{x})\)
3. \((x + y + z)(x + y + \overline{z})(x + \overline{y} + z)(\overline{x} + y + z)\cdot (x + \overline{y} + \overline{z})(\overline{x} + y + \overline{z})(\overline{x} + \overline{y} + \overline{z})\)
4. \(xyz + xy\overline{z} + x\overline{y}z + xy\overline{z} + \overline{x}yz + \overline{x}y\overline{z} + \overline{x}\overline{y}\overline{z}\)