Problem 1. Binary Arithmetic (10 points)

(A) (4 points) What is ~(0xC7) ^ 0x1F, where ~ is bitwise NOT and ^ is bitwise XOR? Provide your result in both binary and hexadecimal. Show your work for partial credit.

Result in binary (0b): 0010 0111
Result in hexadecimal (0x): 27

(B) (3 points) Multiply 9 by 5 using unsigned binary numbers. Show all of your work by filling in the missing rows in the table below and provide your final answer in binary.

\[
\begin{array}{c|c|c|c}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 \\
\end{array}
\]

9 x 5 (0b): 101101

(C) (3 points) What is -41 in 8-bit 2’s complement notation? Provide your answer in binary and hexadecimal.

-41 in binary (using 8-bit 2’s complement notation) (0b): 1101 0111

-41 in hexadecimal (0x): D7
Problem 2. RISC-V Assembly (18 points)

(A) You are given the following C code along with an incomplete translation of this code into RISC-V assembly. Assume that execution ends when it reaches the unimp instruction.

```
int a[4] = {0x1, 0x2, 0x4, 0x8}; // a[0]=0x1, a[1]=0x2, etc.
int b = 0x5;
int c = 0;
for (int i = 3; i >= 0; i = i - 1) {
    c = a[i] & b; // & is bitwise-AND
    if (c != 0){
        break;
    }
}
```

// Translation into RISC-V assembly:
```
    . = 0x0
    li a0, 0x800 // starting address of array
    li a1, 0x5
    li a2, 0x3

loop:  mv t0, a2
    slli t0, t0, 2
    add t0, a0, t0
    lw t1, 0(t0)
    and t1, t1, a1
    bnez t1, end;
    addi a2, a2, -1
    bgez a2, loop
end: unimp
```

1) (4 Points) Complete the RISC-V translation by:
   a. Inserting the loop label where it belongs in the code.
   b. Filling in the blank box with the missing instruction.

2) (6 Points) Provide the values left in the registers below following the execution of the corrected code:

   Value left in t0: 0x808
   Value left in t1: 0x4
   Value left in a2: 0x2
(B) (8 points) Provide the values left in the registers after executing the code below. Assume that execution ends when it reaches the `unimp` instruction. (. = 0x0 means that the first addi instruction is at address 0x0. Zero is x0 which = 0).

```
. = 0x0
    addi a1, zero, 0x234
    addi a3, zero, 0x1
    jal a2, L1
    xor a3, a1, a1
    beqz a3, end
    j L2

L1: addi a2, a2, 4
    jr a2
L2: add a3, a1, a1

end: unimp
```

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<tr>
<th>Value left in a2: 0x10</th>
<th>Value left in a3: 0x468</th>
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</table>

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Value left in a2: 0x10

Value left in a3: 0x468
Problem 3. RISC-V Assembly and Calling Convention (18 points)

(A) (8 points) The \textbf{getPower} function in C given below recursively computes the power of a number (e.g. \texttt{getPower(2,3) = 2^3 = 8}). Since our RISC-V processor does not have a multiply instruction, the code uses a multiply procedure, \texttt{mult}, which multiplies two signed integers. The implementation of the \texttt{mult} procedure is not provided to you, but it does obey the calling convention.

The RISC-V code given below is not behaving as expected and you are tasked with fixing the bugs so that the code works correctly and follows the RISC-V calling convention. You’re allowed to add up to 5 instructions to make it work. You are \textbf{not} allowed to modify or remove any of the given instructions, but feel free to change the order in which they appear.

```c
int getPower(int b, int p){
    if (p == 0)
        return 1;
    else
        return mult(getPower(b, p-1), b);
}
```

Provide the correct assembly code here. Your implementation should obey the RISC-V calling convention.

```assembly
getPower:
    beqz a1, end2
    addi sp, sp, -8
    addi a1, a1, -1
    jal getPower
    jal mult
end2:
    li a0, 1
    ret
end1:
    addi sp, sp, 8
    ret
mult: ...
```

With the correct RISC-V code, what is the value of \texttt{sp} at the pointed instruction (beqz a1, end2) during its third recursive call to \texttt{getPower}? Assume that the original call to \texttt{getPower} is \textbf{not} considered one of the recursive calls. \textbf{Assume sp is 0x101C (in hex) initially.} Assume \( p > 3 \).

\[
\text{sp (0x)} = 0x1004
\]
(B) (10 points) The getSSD function given below computes the sum square distance between the elements in arrays a and b (i.e. getSSD(a[], b[], size) = \sum_{i=0}^{size-1} (a[i] - b[i])^2. Again, this code uses a multiply procedure, mult, which multiplies two signed integers. The implementation of the mult procedure is not provided to you, but it does obey the calling convention.

Please fix the bugs in the given RISC-V code such that it works correctly and follows the RISC-V calling convention. Assume arrays a and b have the same size. You’re allowed to add up to 13 instructions to make it work. Feel free to change the order of instructions, but you are NOT allowed to modify any of the given instructions.

```assembly
int getSSD(int a[], int b[], int size) {
    int sum = 0;
    for (i = 0; i < size; i = i + 1) {
        sum = sum + mult((a[i] - b[i]), (a[i] - b[i]));
    }
    return sum;
}

[] a0 -> base address for a
[] a1 -> base address for b
[] a2 -> size

getSSD:
    mv s1, a0 // a address
    mv s2, a1 // b address
    slli a2, a2, 2
    add a2, a0, a2 // end address of a
    li s0, 0 // s0 holds running sum
    j compare

    lw a4, 0(s1)
    lw a5, 0(s2)
    sub a0, a4, a5
    jal mult
    add s0, s0, a0
    addi s1, s1, 4
    addi s2, s2, 4

    compare:
    bgt a2, s1, loop

end:
    mv a0, s0
    ret
```

Provide the correct assembly code here. Your implementation should obey the RISC-V calling convention. Feel free to only show the changes to the code with arrows to where the changes go in the original code.

```assembly
getSSD:
    addi sp, sp, -20
    sw ra, 0(sp)
    sw s0, 4(sp)
    sw s1, 8(sp)
    sw s2, 12(sp)
    mv s1, a0
    mv s2, a1
    slli a2, a2, 2
    add a2, a0, a2 // get the end address
    li s0, 0 // s0 holds running sum, return 0 if the size <= 0
    j compare

loop:
    lw a4, 0(s1)
    lw a5, 0(s2)
    sub a0, a4, a5
    mv a1, a0
    sw a2, 16(sp)
    jal mult
    lw a2, 16(sp)
    add s0, s0, a0
    addi s1, s1, 4
    addi s2, s2, 4

compare:
    bgt a2, s1, loop

end:
    mv a0, s0
    lw ra, 0(sp)
    lw s0, 4(sp)
    lw s1, 8(sp)
    lw s2, 12(sp)
    addi sp, sp, 20
```
Problem 4. Stack Detective (17 points)

Consider the C procedure below and its translation to RISC-V assembly code, shown on the right.

```c
int f(int a, int b) {
    int c = b - a;
    if (c & 0x11 == 0) { // c is a multiple of 4
        return 1;
    } else {
        int d = f(a - 1, b + 2);
        return 3 * (d + a);
    }
}
```

```assembly
f:    sub a2, a1, a0
       andi a2, a2, 0x11
       bnez a2, ELSE
       li a0, 1
       jr ra  // ret
ELSE: addi sp, sp, -8
       sw a0, 0(sp)
       sw ra, 4(sp)
       addi a0, a0, -1
       addi a1, a1, 2
       jal ra, f
A4:   lw a1, 0(sp)
       lw ra, 4(sp)
L1:    add a0, a0, a1
       slli a1, a0, 1
       add a0, a0, a1
       addi sp, sp, 8
       jr ra  // ret
```

(A) (2 points) What value should the 0x11 term in the C code and the assembly be replaced with to make the if statement correctly check if the variable ‘c’ is a multiple of 4? (*Hint: In the collatz program in Lab 1 we learned that we can test if a number is even, or a multiple of 2, by checking if its last bit is 0.*)

And with 0x11 or 3 to check if last 2 bits are 0 or if multiple of 4

Correct value of ‘0x11’ term? ____3____

(B) (2 points) How many words will be written to the stack before the program makes each recursive call to the function f?

Number of words pushed onto stack before each recursive call? ____2____
The program’s initial call to function $f$ occurs outside of the function definition via the instruction ‘jal ra, f’. The program is interrupted at an execution (not necessarily the first) of function $f$, just prior to the execution of ‘add a0, a0, a1’ at label L1. The diagram on the right shows the contents of a region of memory. All addresses and data values are shown in hex. The current value in the SP register is 0xEB0 and points to the location shown in the diagram.

(C) (4 points) What were the values of arguments $a$ and $b$ to the initial call to $f$? Write CAN’T TELL if the argument does not show up in the stack.

$a$ is passed in a0, can find saved a0 from initial call right before external return address. $b$ is never saved, so can’t tell

Initial arguments to $f$: $a = ____4____$; $b = ____CAN’T TELL____$

(D) (4 points) What are the values in the following registers right when the execution of $f$ is interrupted? Write CAN’T TELL if you cannot tell.

These registers were just loaded from the stack at the time of interruption

Current value of $a1$: 0x______2______

Current value of $ra$: 0x______A4______

(E) (2 points) What is the hex address of the ‘jal ra, f’ instruction that made the initial call to $f$?

Saved ra of initial call is 0x3B8, call occurs 0x4 before that at 0x3B4

Address of instruction that made initial call to $f$: 0x ___3B4____

(F) (3 points) What is the hex address of the instruction at label ELSE?

Ra saved pointing to ‘lw a1, 0(sp)’ is at 0xA4, ELSE is 0x18 before at 0x8C

Address of instruction at label ELSE: 0x ___8C____
Problem 5. Boolean Algebra and Combinational Logic (21 points)

You are given the truth table for a circuit that takes a 3-bit unsigned binary input (X = ABC), adds 3 mod 8 to it to produce a 3-bit unsigned binary output (Y = A’B’C’).

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<tr>
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(A) (9 points) For the above truth table, write out a minimal sum-of-products for each function A’(A,B,C), B’(A,B,C), and C’(A,B,C)

Minimal sum-of-products for $A'(A,B,C)$ = \((\sim A)C + (\sim A)B + A(\sim B)(\sim C)\)

Minimal sum-of-products for $B'(A,B,C)$ = \((\sim B)(\sim C) + BC\)

Minimal sum-of-products for $C'(A,B,C)$ = \(\sim C\)
(B) (3 points) Which one of these functions can be used to build any boolean function (i.e., it’s a universal gate)? Assume that you may tie inputs to 1 or 0 if necessary. **Explain your answer.**

\[ A' \quad B' \quad C' \]

\[ A'(1, B, C) = (\neg B)(\neg C) = (\neg (B + C)) = \text{NOR gate which is universal} \]

(C) (3 points) What function, of A and B, does the following circuit implement?

\[ Y(A, B) = \quad \quad \quad \quad A + B \]

(D) (4 points) Draw the circuit for \( B' \) using only 2-input 1-bit muxes. Make sure to label all the inputs and the output of your circuit.
OR

(E) (2 points) You are told that the propagation delay of a 2-input 1-bit mux is 2ns. What is the propagation delay for output B' based on the circuit you drew above?

\[
\text{Propagation Delay of } B' \text{ (ns)} = \underline{4} \text{ or } 6 \underline{\phantom{0}}
\]
Problem 6. Implementing Combinational Logic (16 points)

(A) (4 points) The following Minispec function \( f \) performs a basic operation using \( a \) and \( b \). We want \( f_2 \) to implement the same function as \( f \). Fill in the blank in \( f_2 \) to make the two functions equivalent. Write a single-line expression that uses the ternary operator (\(? :\) ). Recall that the meaning of an expression like \( \text{return } (x>0)? 1 : 2; \) is if \( x>0 \) then return 1 else return 2.

\[
\text{function } \text{Bit#}(n) \ f#(\text{Integer } n)(\text{Bit#}(n) \ a, \text{Bit#}(1) \ b); \\
\text{Bit#}(n) \ x = 0; \\
\text{for } (\text{Integer } i = 0; i < n; i = i+1) \text{ begin} \\
\text{x[i] = } \neg(\text{a}[i] \mid b) \mid (\text{a}[i] \land b); \\
\text{end} \\
\text{return } x; \\
\text{endfunction}
\]

\[
\text{function } \text{Bit#}(n) \ f2#(\text{Integer } n)(\text{Bit#}(n) \ a, \text{Bit#}(1) \ b); \\
\text{return } (b == 1) ? a : \neg a \\
\text{endfunction}
\]

(B) (6 points) Write the truth table for the combinational device described by this function.

\[
\text{function } \text{Bit#}(2) \ f(\text{Bit#}(1)a, \text{Bit#}(1)b, \text{Bit#}(1)c); \\
\text{Bit#}(8) \ d = 8'hC2 \\
\text{return case}\{a,b}\} \\
0: \{b,c\}; \\
1: (c==0) ? 2'b10 : \{b,a\}; \\
2: 2'b10 ^ \{a,c\}; \\
3: d[7:6]; \\
\text{endcase}; \\
\text{endfunction}
\]

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</table>
(C) (6 points) For the parametric Minispec function g below, select the circuit diagram that correctly implements g when n=2.

```plaintext
function Bit#(n) g#(Integer n)(Bit#(n) a, Bit#(n) b);
    Bit#(n+1) c = 0; // note initial value
    Bit#(n) x = 0;
    for (Integer i = 0; i < n; i = i+1) begin
        x[i] = ~(a[i] ^ b[i]) | c[i];
        c[i+1] = (a[i] & ~b[i]) | (a[i] & c[i]) | (b[i] ^ c[i]);
    end
    return x;
endfunction
```

(c)