6.004 Recitation Problems
L14 – Memory Hierarchy

Keep the most often-used data in a small, fast SRAM (often local to CPU chip). The reason this strategy works: LOCALITY.

- Temporal locality: If a location has been accessed recently, it is likely to be accessed (reused) soon
- Spatial locality: If a location has been accessed recently, it is likely that nearby locations will be accessed soon

AMAT(Average Memory Access Time) = HitTime + MissRatio * MissPenalty

Problem 1. ★

The timing for a particular cache is as follows: checking the cache takes 1 cycle. If there’s a hit the data is returned to the CPU at the end of the first cycle. If there’s a miss, it takes 10 additional cycles to retrieve the word from main memory, store it in the cache, and return it to the CPU. If we want an average memory access time of 1.4 cycles, what is the minimum possible value for the cache’s hit ratio?

Minimum possible value of hit ratio: __________

Problem 2.

The RISC-V Engineering Team is working on the design of a cache. This cache takes 2 clock cycles to determine if a memory access is a hit or a miss and, if it’s a hit, return data to the processor. If the access is a miss, the cache takes 20 additional clock cycles to fill the cache line and return the requested word to the processor. If the hit rate is 90%, what is the processor’s average memory access time in clock cycles?

Average memory access time assuming 90% hit rate (clock cycles): __________

The remainder of this worksheet contains some more pipelining practice in Minispec and a repeat of the questions from R11 about Sequential Logic.
Problem 3. ★

The following Minispec function implements a combinational circuit that adds four 32-bit numbers:

```minispec
typedef Bit#(32) Word;
function Word add4(Vector#(4, Word) x);
    return x[0] + x[1] + x[2] + x[3];
endfunction
```

(A) Draw the maximum-throughput 2-stage pipeline for this circuit.

(B) Implement this 2-stage pipeline as a Minispec module by implementing the rule below. Assume the producer and consumer give and take one input and output every cycle, so no valid bits or stall logic are needed.

```minispec
module PipelinedAdd4;
    RegU#(Vector#(2, Word)) pipeReg1;
    RegU#(Word) pipeReg2;
    input Vector#(4, Word) in;
    method Word out = pipeReg2;
    rule tick;
        pipeReg1 <= in;
        pipeReg2 <= pipeReg1[0]
    endrule
endmodule
```

(C) Complete the skeleton code below to implement a 2-stage pipeline with valid bits (but no stall logic).

```minispec
module PipelinedAdd4;
    Reg#(Maybe#(Vector#(2, Word))) pipeReg1(Invalid);
    Reg#(Maybe#(Word)) pipeReg2(Invalid);
    input Maybe#(Vector#(4, Word)) in default = Invalid;
```

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method Maybe#(Word) out = pipeReg2;

rule tick;

endrule
endmodule
Problem 4. ★

Implement the combination lock FSM from Lecture 10 as a Minispec module. The lock FSM should unlock only when the last four input bits have been 0110. The diagram below shows the FSM’s state-transition diagram.

(A) Implement this state-transition diagram by filling in the code skeleton below. Use the State enum to ensure state values can only be S0-S5.

```verilog
typedef enum { S0, S1, S2, S3, S4 } State;

module Lock;
    Reg#(State) state(S0);
    input Bit#(1) in;
    rule tick;
        state <= case (state)
            S0: ______________________;
            S1: ______________________;
            S2: ______________________;
            S3: ______________________;
            S4: ______________________;
        endcase;
    endrule
    method Bool unlock = ______________________;
endmodule
```
(B) How many flip-flops does this lock FSM require to encode all possible states?

(C) Consider an alternative implementation of the Lock module that stores the last four input bits. Fill in the skeleton code below to complete this implementation.

```verilog
module Lock;
    Reg#(Bit#(4)) lastFourBits(4'b1111);

    input Bit#(1) in;

    rule tick;
        lastFourBits <= _________________________________;
    endrule

    method Bool unlock = _________________________________;
endmodule
```
Problem 5. ★

Implement the Fibonacci FSM from the Sequential Circuits worksheet by filling in the code skeleton below.

From sequential circuits worksheet:
In this problem, we construct a sequential circuit to compute the $N^{th}$ Fibonacci number denoted by $F_N$. The following recurrence relation defines the Fibonacci sequence.

$$F_0 = 0, \ F_1 = 1, \ F_N = F_{N-1} + F_{N-2} \ \forall \ N \geq 2$$

There are two registers $x$ and $y$ that store the Fibonacci values for two consecutive integers. In addition, a counter register $i$ is initialized to $N-1$ and decremented each cycle. The computation stops when register $i$ goes down to 0 and the result ($F_N$) is available in register $x$.

// Use 32-bit values
typedef Bit#(32) Word;

module Fibonacci;
    Reg#(Word) x(0);
    Reg#(Word) y(0);
    Reg#(Word) i(0);

    input Maybe#(Word) in default = Invalid;

    rule tick;
        $i = 0;
        $i <= $i - 1;
        $i;
    endrule

    method Maybe#(Word) result = ________________;
endmodule
Problem 6.

Implement a sequential circuit to compute the factorial of a 16-bit number.

(A) Design the circuit as a sequential Minispec module by filling in the skeleton code below. The circuit should start a new factorial computation when a Valid input is given. Register \( x \) should be initialized to the input argument, and register \( f \) should eventually hold the output. When the computation is finished, the result method should return a Valid result; while the computation is ongoing, result should return Invalid.

You can use the multiplication operator (*) \(*\) performs unsigned multiplication of \(\text{Bit#}(n)\) inputs. Assume inputs and results are unsigned. Lab 5 includes the design of a multiplier from scratch.

```verbatim
module Factorial;
    Reg#(Bit#(16)) x(0);
    Reg#(Bit#(16)) f(0);

    input Maybe#(Bit#(16)) in default = Invalid;

    rule factorialStep;
    endrule

    method Maybe#(Bit#(16)) result =
        ________________________________________;

    endmodule
```

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(B) Manually synthesize your Factorial module into a sequential circuit with registers and combinational logic blocks (similar to how Lecture 11 does this with GCD). No need to draw the implementation of all basic signals (e.g., you can give formulas, like for the sel signal in Lecture 11).