Note: A small subset of essential problems are marked with a red star (★). We especially encourage you to try these out before recitation.

Problem 1. ★

Consider the truth table on the right, which defines two functions F and G of three input variables (A, B, and C).

For each function, write it in normal form, then find a minimal sum of products (minimal SOP) expression.

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<th>F</th>
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</table>

Normal form for F(A,B,C) = ______________________________

Minimal sum of products for F(A,B,C) = ______________________________

Normal form for G(A,B,C) = ______________________________

Minimal sum of products for G(A,B,C) = ______________________________
Problem 2. ★

Consider the 3-input Boolean function \( G(A,B,C) = \overline{A} \cdot \overline{C} + A \cdot \overline{B} + \overline{B} \cdot \overline{C} \)

1. How many 1’s are there in the output column of \( G \)'s 8-row truth table?

2. Give a minimal sum-of-products expression for \( G \).

3. There’s good news and bad news: the bad news is that the stockroom only has \( G \) gates. The good news is that it has as many as you need. Using only combinational circuits built from \( G \) gates, one can implement (choose the best response):
   
   (A) Any Boolean function (\( G \) is functionally complete)  
   (B) Only functions with 3 inputs or less  
   (C) Only functions with the same truth table as \( G \)

4. Can a sum-of-products expression involving 3 input variables with greater than 4 product terms always be simplified to a sum-of-products expression using fewer product terms?
Problem 3. ★

Consider the logic diagram shown below, which includes XOR2, NAND2, NAND3, and INV (inverter) gates.

1. Using the $t_{PD}$ information for the gate components shown in the table above, compute the $t_{PD}$ for the circuit.

\[ t_{PD} = \text{__________} \text{ns} \]

2. Find minimal sum-of-products expressions for both outputs, \( D \) and \( \text{Bout} \).

NOTE: The gates implement the following functions:

- \( \text{NAND2}(a, b) = a \cdot \overline{b} \)
- \( \text{NAND3}(a, b, c) = a \cdot \overline{b} \cdot c \)
- \( \text{XOR2}(a, b) = a \cdot \overline{b} + \overline{a} \cdot b \)

Minimal sum of products for \( D(X,Y,\text{Bin}) = \) __________________________

Minimal sum of products for \( \text{Bout}(X,Y,\text{Bin}) = \) __________________________
Problem 4.

Simplify the following Boolean expressions by finding a *minimal sum-of-products expression* for each one:

1. \( ac + b + c \)
2. \((a + b)c + \bar{c}a + b(\bar{a} + c)\)
3. \(a(\bar{b} + \bar{c})(b + a(b + c))\)
4. \(a(b + c(d + ef))\)
Problem 5.

There are some Boolean expressions for which no assignment of values to variables can produce True (e.g., $a\overline{a}$). Those Boolean expressions are said to be non-satisfiable. Are the following Boolean expressions satisfiable? If the expression is satisfiable, give an assignment to variables that makes the expression evaluate to True. If the expression is non-satisfiable, prove it.

1. $(a + b)c + \overline{c}a + b(\overline{a} + c)$
2. $(x + y)(x + \overline{y})(z + \overline{y})(y + \overline{x})$
3. $(x + y + z)(x + y + \overline{z})(x + \overline{y} + z)(\overline{x} + y + z) \cdot (x + \overline{y} + \overline{z})(\overline{x} + y + \overline{z})(\overline{x} + \overline{y} + \overline{z})$
4. $xyz + xy\overline{z} + x\overline{y}z + xyz + x\overline{y}z + \overline{x}yz + \overline{x}yz$
**Problem 6.**

(A) Simplify the following Boolean expressions by finding a minimal sum-of-products expression for each one. *(Note: These expressions can be reduced into a minimal SOP by repeatedly applying the Boolean algebra properties we saw in lecture.)*

1. \((a + b \cdot \overline{c}) \cdot d + c\)

2. \(a \cdot (b + c)(c + a)\)

(B) There are Boolean expressions for which no assignment of values to variables can produce True (e.g., \(a \cdot \overline{a}\)). These Boolean expressions are said to be *non-satisfiable.*

Are the following Boolean expressions satisfiable? If the expression is satisfiable, give an assignment to variables that makes the expression evaluate to True. If the expression is non-satisfiable, explain why.

1. \((\overline{x} + y\overline{z}) \cdot (\overline{y}x + z) \cdot (\overline{z}y + x)\)

2. \((\overline{x} + y\overline{z}) \cdot (\overline{y}x + z) \cdot (\overline{z}y + x) + (\overline{x} + yz) \cdot (\overline{y}x + z) \cdot (\overline{z}y + x)\)