6.004 Tutorial Problems
L06 – Boolean Algebra and Logic Synthesis

Note: A small subset of essential problems are marked with a red star (★). We especially encourage you to try these out before recitation.

Problem 1. ★

Consider the truth table on the right, which defines two functions F and G of three input variables (A, B, and C).

For each function, write it in **normal form**, then find a **minimal sum of products** (minimal SOP) expression.

A	B	C	F	G
0	0	0	1	1
0	0	1	1	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	1
1	0	1	0	0
1	1	0	0	1
1	1	1	1	0

Normal form for F(A,B,C) = _____

Minimal sum of products for F(A,B,C) = _____

Normal form for G(A,B,C) = _____

Minimal sum of products for G(A,B,C) = _____

Problem 2. ★

Consider the 3-input Boolean function G(A,B,C) = $\bar{A} \cdot \bar{C} + A \cdot \bar{B} + \bar{B} \cdot \bar{C}$

1. How many 1's are there in the output column of G's 8-row truth table?

2. Give a minimal sum-of-products expression for G.

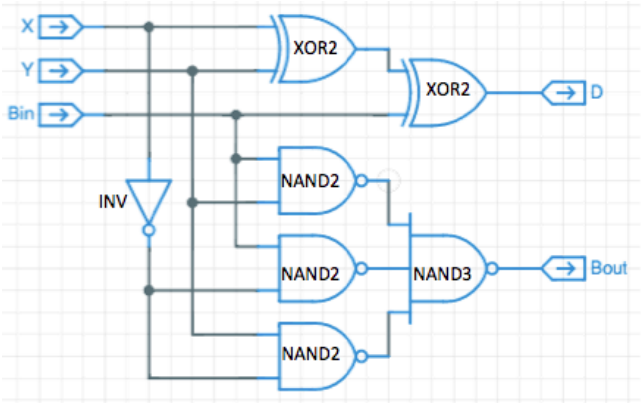
3. There's good news and bad news: the bad news is that the stockroom only has G gates. The good news is that it has as many as you need. Using only combinational circuits built from G gates, one can implement (choose the best response):

   (A) Any Boolean function (G is functionally complete)
   (B) Only functions with 3 inputs or less
   (C) Only functions with the same truth table as G

4. Can a sum-of-products expression involving 3 input variables with greater than 4 product terms *always* be simplified to a sum-of-products expression using fewer product terms?

Problem 3. ★

Consider the logic diagram shown below, which includes XOR2, NAND2, NAND3, and INV (inverter) gates.



Gate	t_{PD}
INV	1.0ns
NAND2	1.5ns
NAND3	1.8ns
XOR2	2.5ns

1. Using the t_{PD} information for the gate components shown in the table above, compute the t_{PD} for the circuit.

t_{PD} = _____ns

2. Find minimal sum-of-products expressions for both outputs, **D** and **Bout**.

   NOTE: The gates implement the following functions:
   - $NAND2(a,b) = \overline{a \cdot b}$
   - $NAND3(a,b,c) = \overline{a \cdot b \cdot c}$
   - $XOR2(a,b) = a \cdot \bar{b} + \bar{a} \cdot b$

Minimal sum of products for D(X,Y,Bin) = _____

Minimal sum of products for Bout(X,Y,Bin) = _____

Problem 4.

Simplify the following Boolean expressions by finding a *minimal sum-of-products expression* for each one:

1. $\overline{ac + b + c}$
2. $(a + b)c + \bar{c}a + b(\bar{a} + c)$
3. $a\overline{(b + c)}(b + a(b + c))$
4. $a(b + c(d + ef))$

Problem 5.

There are some Boolean expressions for which no assignment of values to variables can produce True (e.g., $a\bar{a}$). Those Boolean expressions are said to be *non-satisfiable*. Are the following Boolean expressions satisfiable? If the expression is satisfiable, give an assignment to variables that makes the expression evaluate to True. If the expression is non-satisfiable, prove it.

1. $(a + b)c + \bar{c}a + b(\bar{a} + c)$
2. $(x + y)(x + \bar{y})(z + \bar{y})(y + \bar{x})$
3. $(x + y + z)(x + y + \bar{z})(x + \bar{y} + z)(\bar{x} + y + z) \cdot$
   $(x + \bar{y} + \bar{z})(\bar{x} + y + \bar{z})(\bar{x} + \bar{y} + z)(\bar{x} + \bar{y} + \bar{z})$
4. $\overline{xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}}$

Problem 6.

(A) Simplify the following Boolean expressions by finding a minimal sum-of-products expression for each one. (*Note:* These expressions can be reduced into a minimal SOP by repeatedly applying the Boolean algebra properties we saw in lecture.)

1. $\overline{(a + b \cdot \bar{c})} \cdot d + c$

2. $a \cdot \overline{(b + c)}(c + a)$

(B) There are Boolean expressions for which no assignment of values to variables can produce True (e.g., $a \cdot \bar{a}$). These Boolean expressions are said to be *non-satisfiable*.

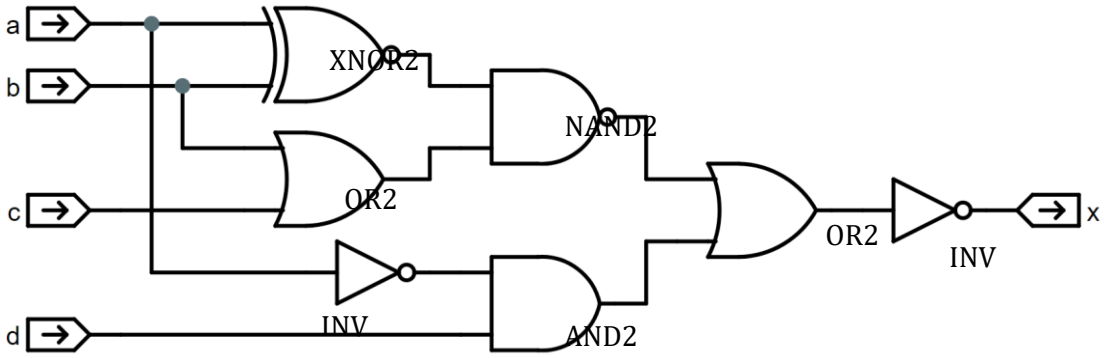
Are the following Boolean expressions satisfiable? If the expression is satisfiable, give an assignment to variables that makes the expression evaluate to True. If the expression is non-satisfiable, explain why.

1. $(\bar{x} + y\bar{z}) \cdot (\bar{y}x + z) \cdot (\bar{z}y + x)$

2. $(\bar{x} + y\bar{z}) \cdot (\bar{y}x + z) \cdot (\bar{z}y + x) + (\bar{x} + yz) \cdot (\bar{y}x + z) \cdot (\bar{z}y + x)$

Problem 7. Boolean Algebra and Combinational Logic (19 points, Spring 2020 Quiz 1)

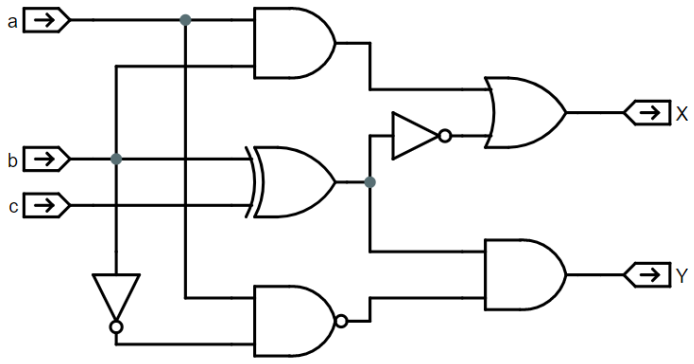
(A) (3 points) Consider the logic diagram below, which includes XNOR2, OR2, NAND2, AND2, and INV. Using the t_{PD} information for the gate components shown in the table below, compute the t_{PD} for the circuit.



Gate	t_{PD}
XNOR2	7.0ns
OR2	5.5ns
NAND2	3.0ns
AND2	5.0ns
INV	2.0ns

t_{PD} (ns) = _____

(B) (6 points) Given the circuit shown below, construct the truth table for outputs **X** and **Y.**



a	b	c	X	Y
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

(C) (4 points) Find a minimal sum-of-products expression for output **X** of the circuit described by the truth table shown below.

a	b	c	d	X
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Minimal sum of products for X = _____

(D) (6 points) For each of the following expressions determine if it is satisfiable. If satisfiable, provide a minimal sum-of-products. Otherwise, show why it is not satisfiable.

1. $\overline{\bar{c}(a+b)(a+d)}(ab\bar{c})$

2. $(x+y)(x\bar{y}z + y\bar{z} + \bar{y})$