Caches
The Memory Hierarchy

Want large, fast, and cheap memory, but...

Large memories are slow (e.g., Hard Disk)
Fast memories are small and expensive (e.g., SRAM)

Solution: Use a hierarchy of memories with different tradeoffs to fake a large, fast, cheap memory
Memory Hierarchy Interface

- Programming model: Single memory, single address space
Memory Hierarchy Interface

- Programming model: Single memory, single address space
- Machine transparently stores data in fast or slow memory, depending on usage patterns
Caches

- Cache: A small, interim storage component that transparently retains (caches) data from recently accessed locations.

- Processor sends accesses to cache. Two options:
  - **Cache hit**: Data for this address in cache, returned quickly.
  - **Cache miss**: Data not in cache.
    - Fetch data from memory, send it back to processor.
    - Retain this data in the cache (replacing some other data).
  - Processor must deal with variable memory access time.
Why Caches Work

- Two predictable properties of memory accesses:
  - **Temporal locality**: If a location has been accessed recently, it is likely to be accessed (reused) soon
  - **Spatial locality**: If a location has been accessed recently, it is likely that nearby locations will be accessed soon

- Result:
  - High hit rate (low miss ratio)
  - Reduced Average Memory Access Time (AMAT):

\[
AMAT = HitTime + MissRatio \times MissPenalty
\]
Basic Cache Algorithm (Reads)

On reference to Mem[X], look for X among cache tags

**HIT:** X = Tag(i) for some cache line i

Return Data(i)

**MISS:** X not found in Tag of any cache line

Read Mem[X]
Return Mem[X]
Select a line k to hold Mem[X]
Write Tag(k) = X, Data(k) = Mem[X]

**Q:** How do we “search” the cache?
Direct-Mapped Caches

- Each word in memory maps into a single cache line
- Access (for cache with $2^w$ lines):
  - Index into cache with $W$ address bits (the index bits)
  - Read out valid bit, tag, and data
  - If valid bit == 1 and tag matches upper address bits, HIT

- Example 8-line direct-mapped cache:

32-bit BYTE address

Valid bit  Tag (27 bits)  Data (32 bits)

0 1 2 3 4 5 6 7

Index bits  Byte offset bits

=?

HIT
Example: Direct-Mapped Caches

64-line direct-mapped cache → 64 indices → 6 index bits

Read Mem[0x400C]

0100 0000 0000 1100

TAG: 0x40
INDEX: 0x3
BYTE OFFSET: 0x0

HIT, DATA 0x42424242

Would 0x4008 hit?
INDEX: 0x2 → tag mismatch → MISS

Part of the address (index bits) is encoded in the location Tag + Index bits unambiguously identify the data’s address
Selection of Index Bits

- Why do we choose low order bits for index?
  - Allows consecutive memory locations to live in the cache simultaneously
  - Reduces likelihood of replacing data that may be accessed again in the near future
  - Helps take advantage of locality
Block Size

- Take advantage of spatial locality: Store multiple words per data line
  - Always fetch entire block (multiple words) from memory
  - Another advantage: Reduces size of tag memory!
  - Potential disadvantage: Fewer indices in the cache
- Example: 4-block, 16-word direct-mapped cache

![Diagram showing the structure of a direct-mapped cache with 4-blocks and 16-words.](image)

- 32-bit BYTE address
- Tag bits: 26 (=32-6)
- Index bits: 2 (4 indices)
- Block offset bits: 2 (4 words/block)
- Byte offset bits: 2

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Block Size Tradeoffs

- Larger block sizes...
  - Take advantage of spatial locality
  - Incur larger miss penalty since it takes longer to transfer the block from memory
  - Can increase the average hit time and miss ratio
- AMAT = HitTime + MissPenalty*MissRatio
Direct-Mapped Cache Problem: Conflict Misses

Assume:
- 1024-line DM cache
- Block size = 1 word
- Consider looping code, in steady state
- Assume WORD, not BYTE, addressing

### Loop A: Code at 1024, data at 37

<table>
<thead>
<tr>
<th>Word Address</th>
<th>Cache Line index</th>
<th>Hit/ Miss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1024</td>
<td>0</td>
<td>HIT</td>
</tr>
<tr>
<td>37</td>
<td>37</td>
<td>HIT</td>
</tr>
<tr>
<td>1025</td>
<td>1</td>
<td>HIT</td>
</tr>
<tr>
<td>38</td>
<td>38</td>
<td>HIT</td>
</tr>
<tr>
<td>1026</td>
<td>2</td>
<td>HIT</td>
</tr>
<tr>
<td>39</td>
<td>39</td>
<td>HIT</td>
</tr>
<tr>
<td>1024</td>
<td>0</td>
<td>HIT</td>
</tr>
<tr>
<td>37</td>
<td>37</td>
<td>HIT</td>
</tr>
</tbody>
</table>

...  

### Loop B: Code at 1024, data at 2048

<table>
<thead>
<tr>
<th>Word Address</th>
<th>Cache Line index</th>
<th>Hit/ Miss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1024</td>
<td>0</td>
<td>MISS</td>
</tr>
<tr>
<td>2048</td>
<td>0</td>
<td>MISS</td>
</tr>
<tr>
<td>1025</td>
<td>1</td>
<td>MISS</td>
</tr>
<tr>
<td>2049</td>
<td>1</td>
<td>MISS</td>
</tr>
<tr>
<td>1026</td>
<td>2</td>
<td>MISS</td>
</tr>
<tr>
<td>2050</td>
<td>2</td>
<td>MISS</td>
</tr>
<tr>
<td>1024</td>
<td>0</td>
<td>MISS</td>
</tr>
<tr>
<td>2048</td>
<td>0</td>
<td>MISS</td>
</tr>
</tbody>
</table>

...  

Inflexible mapping (each address can only be in one cache location) → Conflict misses (multiple addresses map to same cache index)!
Fully-Associative Cache

Opposite extreme: Any address can be in any location

- No cache index!
- Flexible (no conflict misses)
- Expensive: Must compare tags of all entries in parallel to find matching one

32-bit BYTE address

Tag bits

Block offset bits

Byte offset bits

Tag

Valid bit

Data
N-way Set-Associative Cache

- Use multiple direct-mapped caches in parallel to reduce conflict misses
- Nomenclature:
  - # Rows = # Sets
  - # Columns = # Ways
  - Set size = #ways = “set associativity” (e.g., 4-way → 4 lines/set)
- Each address maps to only one set, but can be in any way within the set
- Tags from all ways are checked in parallel

- Fully-associative cache: Extreme case with a single set and as many ways as cache lines

INCOMING ADDRESS

Tag Data

Tag Data

Tag Data

Tag Data

8 sets

4 ways

SET

WAY
Associativity Implies Choices

**Direct-mapped**
- Compare addr with only one tag
- Location A can be stored in exactly one cache line

**N-way set-associative**
- Compare addr with N tags simultaneously
- Location A can be stored in exactly one set, but in any of the N cache lines belonging to that set

**Fully associative**
- Compare addr with each tag simultaneously
- Location A can be stored in any cache line
Replacement Policies

- **Optimal policy**: Replace the line that is accessed furthest in the future
  - Requires knowing the future...
- **Idea**: Predict the future from looking at the past
  - If a line has not been used recently, it’s often less likely to be accessed in the near future (a locality argument)
- **Least Recently Used (LRU)**: Replace the line that was accessed furthest in the past
  - Works well in practice
  - Need to keep ordered list of N items → N! orderings → $O(\log_2 N!) = O(N \log_2 N)$ “LRU bits” + complex logic
  - Caches often implement cheaper approximations of LRU
- **Other policies**:
  - First-In, First-Out (least recently replaced)
  - Random: Choose a candidate at random
    - Not very good, but does not have adversarial access patterns
Write Policy

**Write-through**: CPU writes are cached, but also written to main memory immediately (stalling the CPU until write is completed). Memory always holds current contents

- Simple, slow, wastes bandwidth

**Write-back**: CPU writes are cached, but not written to main memory until we replace the line. Memory contents can be “stale”

- Fast, low bandwidth, more complex
- Commonly implemented in current systems
Example: Cache Write-Hit

- 16-line direct-mapped cache → 4 index bits
- Block size = 4 → 2 block offset bits
- Write Policy = Write Back
- Write: 0x09 to 0x4818

Tag: 0x48
Index: 0x1
Block Off: 2
Byte Off: 0x0

V D Tag (24 bits) Line (4 words, 16 bytes)

D=1: cache contents no longer match main memory so write back line to memory upon replacement
Example: Cache Write-Miss

Write: 0x09 to 0x4818

1. Tags don’t match -> Miss
   - **D=1**: Write cache line 1 (tag = 0x280: addresses 0x28010-0x2801C) back to memory
   - If D=0: Don’t need to write line back to memory.

2. Load line (tag = 0x48: addresses 0x4810-0x481C) from memory

3. Write 0x09 to 0x4818 (block offset 2), set D=1.
Summary: Cache Tradeoffs

\[ AMAT = \text{HitTime} + \text{MissRatio} \times \text{MissPenalty} \]

- Cache size
- Block size
- Associativity
- Replacement policy
- Write policy
Example: Comparing Hit Rates

3 Caches: DM, 2-Way, FA: each has 8 words, block size=1, LRU
Access following addresses repeatedly: 0x0, 0x10, 0x4, 0x24

DM

<table>
<thead>
<tr>
<th>M[0x0]</th>
<th>M[0x4]</th>
<th>M[0x10]</th>
</tr>
</thead>
</table>

2-Way

<table>
<thead>
<tr>
<th>M[0x0]</th>
<th>M[0x10]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M[0x4]</td>
<td>M[0x24]</td>
</tr>
</tbody>
</table>

FA

<table>
<thead>
<tr>
<th>M[0x0]</th>
<th>M[0x10]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M[0x4]</td>
<td>M[0x24]</td>
</tr>
</tbody>
</table>

DM: 50% hit rate
2-Way: 100% hit rate
FA: 100% hit rate
Example 2: Comparing Hit Rates

Access: 0x0, 0x4, 0x8, 0xC, 0x10, 0x14, 0x18, 0x1C, 0x20 repeatedly

<table>
<thead>
<tr>
<th>DM</th>
<th>2-Way</th>
<th>FA</th>
</tr>
</thead>
<tbody>
<tr>
<td>M[0x0] M[0x20]</td>
<td>M[0x0] M[0x20]</td>
<td>M[0x0] M[0x20]</td>
</tr>
<tr>
<td>M[0x4]</td>
<td>M[0x20] M[0x10]</td>
<td>M[0x4] M[0x0]</td>
</tr>
<tr>
<td>M[0x8]</td>
<td>M[0x4] M[0x8]</td>
<td>M[0x8] M[0x4]</td>
</tr>
<tr>
<td>M[0xC]</td>
<td>M[0x8] M[0xC]</td>
<td>M[0xC] M[0x8]</td>
</tr>
<tr>
<td>M[0x10]</td>
<td>M[0xC] M[0x10]</td>
<td>M[0x10] M[0xC]</td>
</tr>
<tr>
<td>M[0x14]</td>
<td>M[0x10] M[0x14]</td>
<td>M[0x14] M[0x10]</td>
</tr>
<tr>
<td>M[0x18]</td>
<td>M[0x14] M[0x18]</td>
<td>M[0x18] M[0x14]</td>
</tr>
<tr>
<td>M[0x1C]</td>
<td>M[0x18] M[0x1C]</td>
<td>M[0x1C] M[0x18]</td>
</tr>
</tbody>
</table>

DM: Hit rate = 7/9  
2-Way: Hit rate = 6/9  
FA: Hit rate = 0%
Example 3: Comparing Hit Rates

Access: 0x0, 0x4, 0x8, 0xC, 0x20, 0x24, 0x28, 0x2C, 0x10 repeatedly

DM

M[0x0]  M[0x20]
M[0x4]  M[0x24]
M[0x8]  M[0x28]
M[0xC]  M[0x2C]
M[0x10]

DM: Hit rate = 1/9

2-Way

M[0x0]  M[0x10]  M[0x20]
M[0x4]  M[0x20]  M[0x10]
M[0x8]  M[0x28]  M[0x20]
M[0xC]  M[0x2C]  M[0x28]

2-Way: Hit rate = 6/9

FA

M[0x0]  M[0x10]
M[0x4]  M[0x0]
M[0x8]  M[0x4]
M[0xC]  M[0x8]
M[0x20]  M[0xC]
M[0x24]  M[0x20]
M[0x28]  M[0x24]
M[0x2C]  M[0x28]

FA: Hit rate = 0%
Thank you!

Next lecture: Operating Systems